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A Class of Unbiased Ratio Estimators

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SUMMARY

A class of unbiased ratio-type estimators for population mean $\overline{Y}$ is defined based on a linear combination of three estimators viz: $\bar{y}$, $(\overline{X}/n)\sum_{i=1}^{n}(y_i/x_i)$ and $(\bar{x}/n)\sum_{i=1}^{n}(y_i/x_i)$. It is shown that Hartley and Ross [1] estimator is a particular member of the class. Exact variance of the proposed class is derived. The optimum estimator in the class in the minimum variance sense is identified. Two numerical examples are included to illustrate the results.

Key words : Unbiased ratio estimator, optimum estimator, minimum variance.

Introduction

Consider a finite population with units $U_1, U_2, \ldots, U_N$. For simplicity let the variate of interest y and the auxiliary variate x related to y assume real non-negative values (y_i, x_i) on the unit U_i, $i = 1, 2, \ldots, N$. We are interested in estimating population mean $\overline{Y}$ utilising information on $\overline{X}$, the population mean of auxiliary character x.

Let y_i and x_i denote respectively the y and x values for the ith sampled unit, $i = 1, 2, \ldots, n$ in simple random sampling without replacement (SRSWOR). Let $r = \bar{y}/\bar{x}$ and $\bar{r} = (1/n)\sum_{i=1}^{n}(y_i/x_i)$ where $\bar{y}$ and $\bar{x}$ are respectively sample means of y and x variates. In SRSWOR Hartley and Ross [1] proposed the following unbiased estimator for $\overline{Y}$:

$$\bar{y}_{HR} = \bar{r}\,\overline{X} + (n(N-1)/N(n-1))\,(\bar{y} - \bar{r}\,\bar{x}) \tag{1.1}$$

or, equivalently

$$\bar{y}_{HR} = \bar{r}\,\overline{X} + ((N-1)/N)\ s_{rx} \tag{1.2}$$

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where, $$s_{rx} = \sum_{i=1}^{n} (r_i - \bar{r})(x_i - \bar{x})/(n-1)$$

In this paper a linear combination of three estimators viz. $\bar{y}, \bar{r}\bar{X}$ and $\bar{r}\bar{x}$ is considered to obtain a class of unbiased ratio estimators for population mean $\bar{Y}$. It is shown that Hartley and Ross [1] estimator belongs to the proposed class. Exact expression for variance of the proposed class is derived. The optimum estimator in the class in the minimum variance sense is identified and the results are illustrated using two numerical examples.

2. *A Class of Unbiased Ratio-Type Estimators in SRSWOR*

We propose a class of ratio-type estimators for $\bar{Y}$ as

$$\hat{\bar{Y}} = \theta_1 \bar{y} + \theta_2 \bar{r}\bar{X} + \theta_3 \bar{r}\bar{x} \tag{2.1}$$

where θ_i's are suitably chosen constants such that $\sum_{i=1}^{3} \theta_i = 1$.

The estimator $\hat{\bar{Y}}$ is unbiased for $\bar{Y}$ if $B(\hat{\bar{Y}}) = 0$.

i.e., $$\theta_2 B(\bar{r}\bar{X}) + \theta_3 B(\bar{r}\bar{x}) = 0$$

where $B(\cdot)$ stands for bias of $(\cdot)$

or $$\theta_2 = \theta_3 \frac{B(\bar{r}\bar{x})}{B(\bar{r}\bar{x})} \tag{2.2}$$

Now, it can be shown that

$$B(\bar{r}\bar{X}) \doteq -[\frac{(N-1)}{N}] S_{rx} \tag{2.3}$$

and $$B(\bar{r}\bar{x}) = -[\frac{(n-1)}{n}] S_{rx} \tag{2.4}$$

where $$S_{rx} = \sum_{i=1}^{N}(r_i - \bar{R})(x_i - \bar{X})/(N-1)$$

and $$\bar{R} = (1/N)\sum_{i=1}^{N} r_i = \sum_{i=1}^{N} (y_i/x_i)/N \tag{2.4}$$

It follows from (2.3) and (2.4) that

$$\frac{B(\bar{r}\,\bar{x})}{B(\bar{r}\,\bar{X})} = \frac{N(n-1)}{n(N-1)} \tag{2.5}$$

Substituting (2.5) in (2.2) we get

$$\theta_2 = -\frac{N(n-1)}{n(N-1)}\theta_3$$

As $\sum_{i=1}^{3} \theta_i = 1$, it follows that

$$\theta_1 = 1 - \frac{(N-n)}{n(N-1)}\theta_3$$

Letting $\theta_3 = \theta$ (a constant) and substituting the values of θ_i's $(i = 1, 2, 3)$ in (2.1), we get a class of unbiased ratio-type estimators given by

$$\hat{Y}_u = \left[1 - \frac{(N-n)\theta}{n(N-1)}\right]\bar{y} - \left[\frac{N(n-1)}{n(N-1)}\right]\theta\bar{r}\,\bar{X} + \theta\,\bar{r}\,\bar{x} \tag{2.6}$$

or equivalently

$$\hat{Y}_u = [1 - (1-c)\,\theta\,]\,\bar{y} - c\,\theta\,\bar{r}\,\bar{X} + \theta\bar{r}\,\bar{x} \tag{2.7}$$

where $c = \frac{N(n-1)}{n(N-1)}$

Different estimators can be generated from (2.6) for suitably chosen value of θ. Some such estimators are listed below

(i) For $\theta = -\frac{n(N-1)}{N(n-1)}$. $\hat{Y}_u$ gives

$$\hat{Y}_1 = \bar{r}\,\bar{X} + \frac{n(N-1)}{N(n-1)}(\bar{y} - \bar{r}\,\bar{x})$$

This estimator is due to Hartley and Ross [1].

(ii) For $\theta = \frac{n(N-1)}{(N-n)}$, $\hat{Y}_u$ turns into

$$\hat{Y}_2 = \frac{n(N-1)}{(N-n)}\bar{r}\,\bar{x} - \frac{N(n-1)}{(N-n)}\bar{r}\,\bar{X}$$

(iii) For $\theta = 1, \hat{Y}_u$ becomes

$$\hat{Y}_3 = \frac{N(n-1)}{n(N-1)}(\bar{y} - \bar{r}\bar{X}) + \bar{r}\bar{x}$$

(iv) For $\theta = -\frac{n(N-1)}{(N-n)}, \hat{Y}_u$ gives

$$\hat{Y}_4 = 2\bar{y} + \frac{N(n-1)}{(N-n)}\bar{r}\bar{X} - \frac{n(N-1)}{(N-n)}\bar{r}\bar{x}$$

(v) For $\theta = -\frac{n(N-1)}{N(n-1)}, \hat{Y}_u$ tends to

$$\hat{Y}_5 = \frac{(Nn-2N+n)}{N(n-1)}\bar{y} + \frac{n(N-1)}{N(n-1)}\bar{r}\bar{x} - \bar{r}\bar{X}$$

(vi) For $\theta = -1$, we get

$$\hat{Y}_6 = \frac{N(n-1)}{n(N-1)}(\bar{y} - \bar{r}\bar{X}) - \bar{r}\bar{x}$$

3. *Optimum Estimator in the Class* $\hat{Y}_u$

We have from (2.7) that

$$V(\hat{Y}_u) = [1-(1-c)\theta]^2 V(\bar{y}) + c^2\theta^2 V(\bar{r}\bar{X}) + \theta^2 V(\bar{r}\bar{x})$$

$$-2[1-(1-c)\theta]c\theta \operatorname{Cov}(\bar{y}, \bar{r}\bar{X}) + 2[1-(1-c)\theta]\theta \operatorname{Cov}(\bar{y}, \bar{r}\bar{x})$$

$$-2c\theta^2 \operatorname{Cov}(\bar{r}\bar{X}, \bar{r}\bar{x}) \tag{3.1}$$

$$= (1+\theta c)^2 V(\bar{y}) + \theta^2 c^2 V(\bar{y}_{HR}) - 2(1+\theta c)\theta c \operatorname{Cov}(\bar{y}, \bar{y}_{HR})$$

$$= \theta^2 c^2 [V(\bar{y}) + V(\bar{y}_{HR}) - 2\operatorname{Cov}(\bar{y}, \bar{y}_{HR})]$$

$$+ 2\theta c[V(\bar{y}) - \operatorname{Cov}(\bar{y}, \bar{y}_{HR})] + V(\bar{y}) \tag{3.2}$$

where $V(\cdot)$ stands for variance of $(\cdot)$

In SRSWOR, the exact expressions for variance and covariances involved in (3.2) are respectively given by,

$$V(\bar{y}) = \frac{(N-n)}{n(N-1)}\sigma_y^2 = (1-c)\sigma_y^2 \tag{3.3}$$

$$V(\bar{y}_{HR}) = \bar{x}^2 V(\bar{r}) + \left(\frac{(N-1)}{N}\right)^2 V(s_{rx}) + \left[\frac{N-1}{N}\right]\bar{X}\operatorname{Cov}(r, s_{rx}) \tag{3.4}$$

where $V(\bar{r}) = (1-c)\,\sigma_r^2$

$$V(s_{rx}) = (1-c)\,A_1\,\mu_{220}(r,x,y) + A_2\,\mu_{200}(r,x,y)\,\mu_{020}(r,x,y) - A_3\,\mu_{110}^2(r,x,y)$$

$$A_1 = \frac{N(Nn - N - n - 1)}{(n-1)(N-2)(N-3)}$$

$$A_2 = \frac{N(N - n - 1)}{(n-1)(N-2)(N-3)}$$

$$A_3 = \frac{(N^2 n - nN - 2N^2 + 4N - 2n - 2)N}{(n-1)(N-1)(N-2)(N-3)}$$

$$\text{Cov}(\bar{r}, s_{rx}) = (1-c)\left(\frac{N}{N-2}\right)\mu_{210}(r,x,y)$$

$$\text{Cov}(\bar{y}, \bar{y}_{HR}) = \bar{X}\,\text{Cov}(\bar{r}, \bar{y}) + \left(\frac{N-1}{N}\right)\text{Cov}(\bar{y}, s_{rx})$$

$$\text{Cov}(\bar{r}, \bar{y}) = (1-c)\,\mu_{101}(r,x,y)$$

$$\text{Cov}(\bar{y}, s_{rx}) = (1-\bar{X})\left(\frac{N}{N-2}\right)\mu_{111}(r,x,y)$$

where, $\mu_{abe}(r,x,y) = \sum (\bar{r}_i - R)^a (\bar{x}_i - X)^b (\bar{y}_i - Y)^e / N$

(a, b, e are non-negative integers)

The explicit expression for the variance of $\hat{Y}_u$ in terms of θ and c can be obtained by substituting the expressions from (3.3) to (3.5) in (3.2). The variance of Y_u given in (3.2) will be minimum when

$$\theta = -(1/c)\left[\frac{V(\bar{y}) - \text{Cov}(\bar{y}, \bar{y}_{HR})}{V(\bar{y}) + V(\bar{y}_{HR}) - 2\text{Cov}(\bar{y}, \bar{y}_{HR})}\right]$$

$$= -(1/c)\left[\frac{\text{Cov}(\bar{y}, d)}{V(d)}\right] = \theta_o \quad \text{(say)} \qquad (3.6)$$

where, $d = (\bar{y} - \bar{y}_{HR})$

Hence, the minimum variance of $\hat{Y}_u$ is given by

$$V(\hat{Y}_u^{opt}) = V(\bar{y}) - \frac{[\text{Cov}(\bar{y}, d)]^2}{V(d)}$$

$$= V(\bar{y})\,[1 - \rho_1^2] \tag{3.7}$$

where, $$\rho_1 = \frac{\text{Cov}\,(\bar{y}, d)}{\sqrt{V\,(\bar{y})\,V(d)}}$$

It is obvious from (3.7) that

$$V(\hat{Y}_u^{opt}) < V(\bar{y})$$

It can also be shown that

$$V(\bar{y}_{HR}) - V(\hat{Y}_u^{opt}) = \frac{\{V(\bar{y}_{HR}) - \text{Cov}\,(\bar{y}, \bar{y}_{HR})\}^2}{V(d)} \geq 0$$

Hence, $\hat{Y}_u^{opt}$ is more efficient than $\bar{y}_{HR}$. Thus $\hat{Y}_u$ is more efficient than both $\bar{y}$ and $\bar{y}_{HR}$ when selected θ coincides with optimum θ *i.e.*, θ_o.

In case θ does not coincide with its optimum value, then it can be shown that $\hat{Y}_u$ is more efficient than $\bar{y}$ if θ is selected such that

either $\quad 2\theta_o < \theta < 0 \quad$ when $\theta_o < 0$

or $\quad \theta < \theta < 2\theta_o \quad$ when $\theta_o > 0$ (3.8)

Likewise it can be shown that

$$V(\hat{Y}_u) = V(\bar{y}_{HR}) \text{ if}$$

either $\quad -(1/c) < \theta < 2\theta_o + (1/c)$ if $\theta_o > -(1/c)$

or $\quad 2\theta_o + (1/c) < \theta < -(1/c)$ if $\theta_o < -(1/c)$ (3.9)

where θ_o is as defined in (3.6)

The expression for θ written in (3.6) can approximately be expressed in terms of bivariate population moments as

$$\theta = \frac{[(N-1)/(N-2)]\{\bar{X}\,\mu_{12} - R\bar{X}\,\mu_{21} - \mu_{22} + \mu_{11}^2\} - \bar{X}^2 R\mu_{11} - \bar{X}\,\mu_{12}}{c\left[R^2\bar{X}^2\,\mu_{20} + \mu_{22} + \mu_{11}^2 + 2R\bar{X}\,\mu_{21} + \left(\frac{N-1}{N}\right)^2 (A_1D_1 + A_2D_2 - A_3D_3)^2 + 2\frac{N-1}{N-2}D_4\right]} \tag{3.10}$$

where

$$D_1 = [\mu_{22} + R^2\,\mu_{40} - 2R\,\mu_{31}]$$

$$D_2 = [\mu_{02} + R^2 \mu_{20} + (\mu_{22}/\overline{X}^2) + (\mu_{11}/\overline{X}^2) - 2R\,\mu_{11} - 2(\mu_{12}/\overline{X}) + 2R(\mu_{21}/\overline{X})]\,\mu_{20}$$

$$D_3 = [\mu_{11} - R\,\mu_{20} - (\mu_{21}/\overline{X})]$$

$$D_4 = [R^2\,\overline{X}\,\mu_{30} - R\,\overline{X}\,\mu_{21} - \mu_{22} + \mu_{11}^2 + 2R\,\mu_{31} - 2R\,\mu_{20}\,\mu_{11}]$$

Remark : It is apparent from (3.6) and (3.10) that computation of θ requires the values of different population parameters, which in practice will be rarely known. It is however, possible in repeated surveys based on multiphase sampling, where information on the same set of characters is collected over several occasions to guess the values of these parameters and hence of θ. However, if no such information is available, these parameters can be estimated by their respective consistent estimators and value of θ computed. Even if this value differs from optimum value, the proposed class of estimators is expected to be more efficient than $\overline{y}$ and $\overline{y}_{HR}$ subject to (3.8) and (3.9) respectively.

4. *Empirical performance of* $\hat{Y}_u$

For the purpose of illustration, SRSWOR is assumed throughout this section. The relative efficiency (RE) of $\hat{Y}_u$ is empirically assessed for two populations one of which (population I) was studied by Srivenkataramana and Tracy [3] and the other (population II) is a population of live data taken from Singh *et al.* ([2], p 139).

First consider population I (see Srivenkataramana and Tracy [3], Table 7.1) of size 7. The required parameters for population I are

$$\overline{Y} = 5.00,\ \overline{X} = 6.00,\ \overline{R} = 1.01,\ \mu_{110}(r, x, y) = -1.05,\ \mu_{101}(r, x, y) = 0.85$$

$$\mu_{002}(r, x, y) = 7.43,\ \mu_{020}(r, x, y) = 6.00,\ \mu_{200}(r, x, y) = 0.54$$

$$\mu_{210}(r, x, y) = -1.34,\ \mu_{220}(r, x, y) = 5.41,\ \mu_{111}(r, x, y) = -0.56$$

Computations were done for n = 2. The table 4.1 gives the variance and RE of $\hat{Y}_u$ with respect to four estimators *viz.* : $\overline{y}$ (sample mean), $\overline{y}_r = \overline{y}\,(\overline{X}/\overline{x}) = r\,\overline{X}$ (usual ratio estimator), $\overline{y}_r' = \overline{r}\,\overline{X}$ (another ratio estimator) and $\overline{y}_{HR}$ (Hartley and Ross, [1]) for different values of θ. In computations the exact expressions for variances, covariances or mean square errors (mse) as the case may be were used for $\overline{y}, \overline{y}_r'$ and $\overline{y}_{HR}$. The mse of $\overline{y}_r$ was computed using second order approximation as per Sukhatme *et al.* [4]. The maximum gain in efficiency was observed over all the estimators studied when the θ value corresponds to the optimum value of $\theta = -0.70$. The estimator $\hat{Y}_u$ is efficient as compared

Table 4.1. Variance and RE of $(\hat{Y}_u)$ over different estimators for population I

θ	$V(\hat{Y}_u)$	RE over			
		$\bar{y}$	$\bar{y}_r$	$\bar{y}_r'$	$\bar{y}_{HR}$
0	3.10	100.00	119.03	262.26	118.07
−0.1	2.96	104.73	124.66	274.66	123.65
−0.2	2.84	109.16	129.93	286.27	128.87
−0.3	2.75	112.73	134.18	295.64	133.09
−0.4	2.68	115.67	137.69	303.36	136.57
−0.5	2.63	117.87	140.30	310.31	139.16
−0.6	2.60	119.23	141.92	312.69	140.77
−0.7	2.59	119.69	142.47	313.90	141.31
−0.8	2.60	119.23	141.92	312.69	140.77
−0.9	2.63	117.87	140.30	309.13	139.16
−1.0	2.68	115.67	137.69	303.36	136.57
−1.1	2.76	112.32	133.70	294.57	132.61
−1.2	2.85	108.77	129.47	285.26	128.42
−1.3	2.97	104.38	124.24	273.74	122.41
−1.4	3.10	100.00	119.03	262.26	118.07
$\hat{Y}_1$ ($\theta = -1.71$)	3.66	84.67	100.82	222.13	100.00
$\hat{Y}_2$ ($\theta = 2.40$)	12.60	24.60	29.29	64.52	29.05
$\hat{Y}_3$ ($\theta = 1.00$)	5.59	55.46	66.01	145.44	65.47
$\hat{Y}_4$ ($\theta = -2.40$)	5.61	55.26	65.78	144.92	65.24
$\hat{Y}_5$ ($\theta = 1.71$)	8.64	35.88	42.71	94.10	42.36
$\hat{Y}_6$ ($\theta = 1.00$)	2.68	115.67	137.69	303.36	136.57

$V(\hat{Y}) = 3.10$ $\quad MSE(\bar{y}_r) = 3.69$ $\quad MSE(\bar{y}_r') = 8.13$ $\quad V(\bar{y}_{HR}) = 3.66$

to all the four estimators even when θ deviates in the neighbourhood of its optimum value the range being from 0 to −1.4. It is interesting to note that

$\hat{Y}_6$ a member of $\hat{Y}_u$ performs better than the other estimators studied including Hartley and Ross [1] estimator.

We now turn our attention to population II (Singh *et al.*, [2], p. 139) of size 13. The variates considered are

y = Total number of guava trees ('00).

x = Area under guava orchard (acres).

The required parameters for population II are

$\bar{Y} = 7.47,\ \bar{X} = 5.66,\ \bar{R} = 1.48,\ \mu_{110}\ (r, x, y) = -0.88,\ \mu_{101}\ (r, x, y) = -0.48$

Table 4.2. Variance and RE of ($\hat{Y}_u$) over different estimators for population II

θ	$V(\hat{Y}_u)$	RE over			
		$\bar{y}$	$\bar{y}_r$	$\bar{y}_r'$	$\bar{y}_{HR}$
–0.6	1.80	288.89	77.22	80.56	100.56
–0.7	1.52	342.11	91.45	95.39	119.08
–0.8	1.33	390.98	104.51	109.02	136.09
–0.9	1.21	429.75	114.88	119.83	149.59
–1.0	1.18	440.68	117.80	122.88	153.39
–1.1	1.23	422.76	113.01	117.89	147.15
–1.2	1.36	382.35	102.21	106.62	133.09
–1.3	1.57	331.21	88.54	92.36	115.29
–1.4	1.86	279.57	74.73	77.96	97.31
$\hat{Y}_1$ ($\theta = -1.39$)	1.81	287.29	76.80	80.11	100.00
$\hat{Y}_2$ ($\theta = 3.60$)	87.48	5.94	1.59	1.66	2.07
$\hat{Y}_3$ ($\theta = 1.00$)	17.41	29.87	7.98	8.33	10.40
$\hat{Y}_4$ ($\theta = -3.60$)	29.05	17.90	4.79	4.99	6.23
$\hat{Y}_5$ ($\theta = 1.39$)	24.39	21.32	5.70	5.95	7.42
$\hat{Y}_6$ ($\theta = -1.00$)	1.18	440.68	117.80	122.88	153.39

$V(\hat{Y}) = 5.20$ $\quad MSE_2(\bar{y}_r) = 1.39$ $\quad MSE(\bar{y}_r') = 1.45$ $\quad V(\bar{y}_{HR}) = 1.81$

$$\mu_{002}(r, x, y) = 18.71, \ \mu_{020}(r, x, y) = 12.50, \ \mu_{200}(r, x, y) = 0.16$$

$$\mu_{210}(r, x, y) = -0.10, \ \mu_{220}(r, x, y) = 1.90, \ \mu_{111}(r, x, y) = 1.08$$

Computations were done for n = 3. The table 4.2 shows the variance and RE of Y_u over other four estimators considered in the study. The variances or mse's were computed as mentioned for population I. The maximum gain in efficiency over all the estimators is observed when θ assumes the optimum value of $\theta = -1.00$. The estimator Y_u is seen to be more efficient as compared to $\overline{Y}_{HR}$ even when θ departs in the neighbourhood of its optimum, the range being -0.6 to -1.38. The common range of θ values in which the estimator Y_u is superior to both $\bar{y}_r$ and $\bar{y}_r'$ is comparatively smaller ranging from -0.77 to -1.22. It is to be noted that the estimator $\hat{Y}_6$ a member of $\hat{Y}_u$ which coincides with the optimum estimator, recorded maximum gain in efficiency as compared to all other estimators.

Empirical study thus demonstrates that the survey practitioner can generate estimators which are more efficient than sample mean as well as ratio and Hartley and Ross [1] estimators from the proposed class of estimators Y_u for certain optimum or near optimum values of θ. It is possible in practice to obtain a near optimum value of θ using (3.6) or (3.10) by substituting the values of estimated variances and covariances from the data at hand.

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A New Predictive Ratio Estimator

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SUMMARY

Using the predictive approach advocated by Basu [1] we develop an almost unbiased ratio estimator of a finite population mean which is found to be more efficient than its competitors.

Key words : Almost unbiased ratio estimator, efficiency, mean square error, predictive approach.

Introduction and construction of the new predictive ratio estimator

Let y_i and x_i $(1 \leq i \leq N)$ be the values of two positively correlated variates y and x defined on a finite population of N units with means $\overline{Y}$ and $\overline{X}$ respectively. Let $(\overline{y}, \overline{x})$ and $(\overline{Y}_r, \overline{X}_r)$ denote respectively the means over a simple random without replacement samples of n units and the unsampled residuum. Under the usual prediction approach of Basu [1] an estimator of $\overline{Y}$ is given by

$$\hat{\overline{Y}} = \frac{n}{N}\overline{y} + \frac{N-n}{N}T$$

where T is the implied predictor of $\overline{Y}_r$. If we use $T_R = \overline{y}\,\overline{X}_r/\overline{x}$ as a predictor of $\overline{Y}_r$, $\hat{\overline{Y}}$ reduces to the classical ratio estimator $\hat{\overline{Y}}_R = \overline{y}\,\overline{X}/\overline{x}$ (cf. Srivastava [3]).

Using Taylor linearization method (e.g. Cochran [2]) and noting that to $O(n^{-1})$,

$$E(T_R) = \overline{Y}\left[1 + \frac{N}{N-n}\left(\frac{V(\overline{x})}{\overline{X}^2} - \frac{Cov(\overline{x}, \overline{y})}{\overline{X}\,\overline{Y}}\right)\right]$$

an almost unbiased ratio estimator of $\overline{Y}$ can be obtained as

$$T_{MR} = T_R\left[1 + \theta\frac{N}{N-n}(c_{xy} - c_x^2)\right],$$

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